

Information spreading on dynamic social networks

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Abstract. - Nowadays, information spreading on social networks has triggered an explosive attention in various disciplines. Most of previous related works in this area mainly focus on discussing the effects of spreading probability or immunization strategy on static networks. However, in real systems, the peer-to-peer network structure changes constantly according to frequently social activities of users. In order to capture this dynamical property and study its impact on information spreading, in this Letter, a link rewiring strategy based on the Fermi function is introduced. In the present model, the informed individuals tend to break old links and reconnect to ones with more uninformed neighbors. Simulation results on the susceptible-infected (*SI*) model with non-redundancy contacts indicate that the information spread more faster and broader with the rewiring strategy. Extensive analyses of the information cascading show that the spreading process of the initial steps plays a very important role, that is to say, the information can spread out if it can survive after the beginning time. The proposed model may shed some light on the in-depth understanding of information spreading on dynamical social networks.

Introduction. – The epidemic spreading based on complex networks, where nodes represent individuals or organizations and links denote their interactions, has attracted an increasing attention in recent years [1–3]. Epidemic spreading is a dynamic process in which an item is transmitted from an infected individual to a susceptible individual through the link between them. Therefore, the network structure is a particularly important factor for the efficiency of epidemic spreading. Recently, many pioneering works about susceptible-infected-susceptible (SIS) and susceptible-infected-removed (SIR) models indicate that a highly heterogeneous structure will lead to the absence of any epidemic threshold [4] while the epidemic spreading on small-world network exhibits critical behavior [5]. The voluntary vaccination strategy under game theory framework shows that the epidemic spreading on scale-free networks can be favorably and easily controlled [6]. However, all these interesting results are obtained based on the research of the static network, where interactions are fixed. By contrast, in real online systems, people communicate with various individuals and make new friends everyday.

That is to say, the social communication network, also referred to as the peer-to-peer network, would change its topology dynamically. Consequently, it is very suitable to study such dynamic networks with the *rewiring strategy* [7], where the network structure changes with breaking old links and reforming new ones.

In the past few years, many researches have focused on the epidemic spreading problem on such dynamically contacting networks based on the link rewiring strategy [8–14]. The most important and widely used one is the *adaptive model* [9, 10], in which the susceptible individuals try to avoid contacting with the infected individuals [9, 11, 12]. Simulation results of SIS [9, 12] and susceptible-infected-recovery-susceptible (SIRS) models [11] on adaptive networks show that the epidemic threshold becomes larger than that on static networks. It indicates that the rewiring process typically tends to suppress epidemic spreading, for the infected individuals become isolated with the rewiring method. Recently, Yoo *et al* [13] introduced the *fitness-adaptive rewiring* model where each individual's degree is preserved based on adaptive model [14]. The reaching epidemic threshold is delayed and the prevalence is reduced comparing with the adaptive model.

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els. All the epidemic spreading researches on dynamic networks based on the adaptive model indicate that isolating infected individuals (or susceptible individuals) is an efficient strategy of reducing the fraction of susceptible-infected interactions and preventing the outbreak of the whole spreading process.

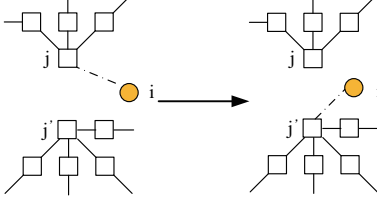


Fig. 1: (Color online) Illustration of the rewiring process. The left panel is the original network structure, and the filled circle (i) represents the informed individual, while square symbols represent the uninformed individuals. The payoffs of j and j' are $\pi_j = 3$ and $\pi_{j'} = 4$ respectively. Node i will break the link to j (left-hand panel) and connect to j' (right-hand panel) with probability $p_f = \frac{1}{1 + e^{\beta(\pi_j - \pi_{j'})}}$.

However, the information spreading is very different from the disease infections due to its specific features, such as the time decaying effect [15], tie strength [16], information contents [17], role of spreaders, memory effects [18], social reinforcement [19, 20] and non-redundancy of contacts [21]. In this Letter, we proposed a new rewiring model to study the information spreading on dynamic networks where individuals will select the neighbors with better payoff [22] following the Fermi function [23–25]. Different from adaptive models, in this model active infected individuals will break the susceptible-infected link if the susceptible individual's payoff (the susceptible neighbors of the considering individual) is less than the randomly chosen one, and rewire the link to the selected susceptible individual (Fig. 1). Simulation results on various networks show that the spreading on dynamic networks is more efficient than the static networks. Especially for the scale-free networks, the information spread prevalence forms two regimes, that is the information diffusion either dies out quickly or spreads out to a fraction of the total population.

Model. — In the information spreading process, when an individual have informed the news (or the signals), s/he can't be back to the uninformed state unless s/he have forgotten the information completely. Generally speaking, the forgetting time-scale should be much larger than the information spreading time-scale. As a result, in this Letter, we describe the information spreading process with the susceptible-infected (SI) model, in which individuals must be two discrete states, that s/he either knows or doesn't know the information and the unknown individuals can be informed to know it while the informed individuals cannot be uninformed state any more. We define I as the informed state, and S as uninformed case.

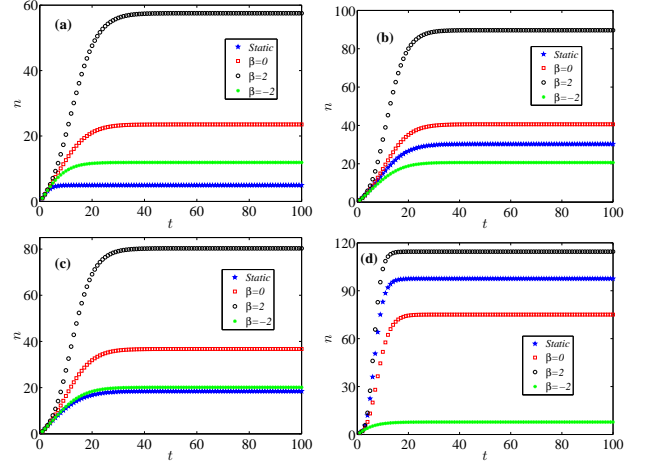


Fig. 2: (Color online) The dynamic of the informed individuals number with different methods including static network (blue pentagram), $\beta = 0$ (red square), $\beta = 2$ (black cycle) and $\beta = -2$ (green star). (a) regular network; (b) random network; (c) small-world network; (d) scale-free network.

Initially, a node is randomly chosen as the I state which is used as the *seed* for the information spreading, and all other individuals are in the S state that haven't received the information yet. At each time step, the I node transmits the information to the S nodes through SI links with the spreading rate λ . In social network, the individuals usually don't transfer an information item more than once to the same one, namely as the non-redundancy of contacts [21]. Therefore, each SI link just can be used once in our model, no matter the transmission through this link is successful or not. When the active I node, which is also the newly informed node, transmits the information to all its S neighbors at one time step, the I node will be inactive and doesn't transmit information any more. And the SI links correspond to the active I nodes are the active SI links (the dash dot line in Fig. 1).

Generally, information decays very fast [15], that is to say, some information would lose attraction in a short period, hence how to spread the information quickly and broaden the use of information is a critically important problem in the social system [26]. The link rewiring strategy is possible to enhance the information spreading efficient through changing the network structure. As a result, we consider the link rewiring strategy as the following method (see Fig. 1). For each active SI link (denote the corresponding S node as j), I node rewires the link to a randomly chosen S node (j') with probability p_f , which is chosen according to the Fermi function [23–25] from statistical physics:

$$p_f = \frac{1}{1 + e^{\beta(\pi_j - \pi_{j'})}}, \quad (1)$$

where $\pi_j, \pi_{j'}$ are respectively the payoffs of two S -state nodes s_1, s_2 . The payoff is defined as the number of the S nodes that connected with the corresponding node. In

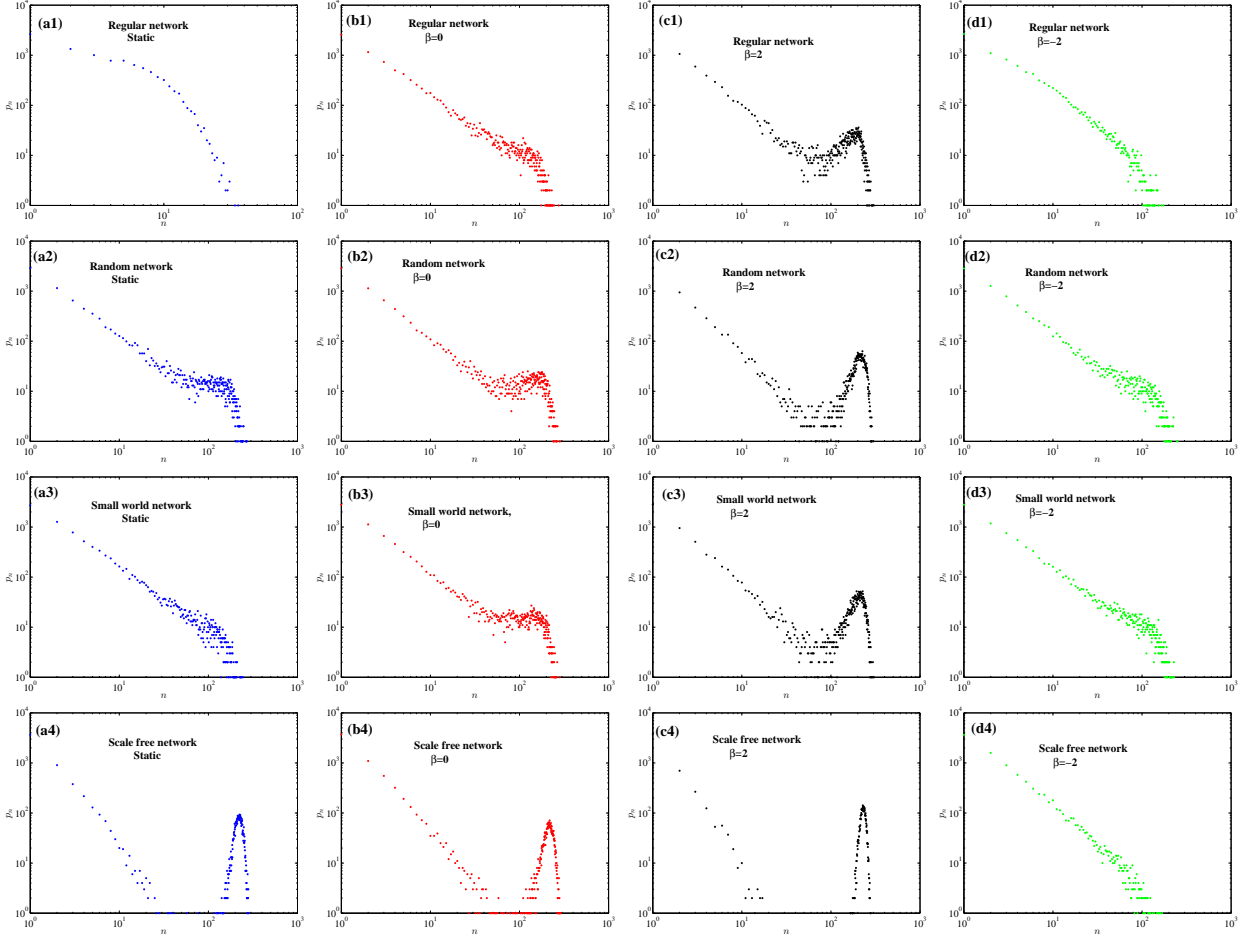


Fig. 3: (Color online) The cascade size distribution p_n with randomly select the spreading “seed” for different networks. The figures from left to right are simulated with static network, $\beta = 0$, $\beta = 2$ and $\beta = -2$ respectively; from top to bottom are regular networks, random networks, small world networks and scale free networks respectively. The results are obtained by averaging over 10000 independent realizations.

general, the individual with more connections is regarded as *information hungry*, for they will be more likely to communicate with others.

The parameter β , which is inversely proportional to the absolute temperature in the statistical physics, controls the intensity of the rewiring probability. In our model, β is generalized to $[-\infty, +\infty]$. The node with larger payoff will be chosen with larger probability when $\beta > 0$, and vice verse. And the rewiring probability becomes neutral when $\beta = 0$, which corresponds to the case of random rewiring.

In the traditional *SI* model, the information will spread until all individuals receive the information. However, with considering the non-redundancy contacts, the spreading process will stop when there is no active *SI* link in the whole system.

Result. — The proposed model is performed on four representative networks with the same total population N and average degree k . 1) *regular network*: a ring lattice with N nodes and k edges per node [27]; 2) *small-world*

network: rewiring each edge at random with probability p_s based on the regular network [27]; 3) *random network*: randomly rewiring probability $p_s = 1$ [27]; and 4) *scale-free network*: $m = k/2$ in the BA model where m is the number of edges for the new node [28], and the network exhibits a power-law degree distribution $p(k) \sim k^{-\gamma}$ with $\gamma = 3$. For the *seed* will impact the spreading process seriously, all the simulation results are obtained by averaging over 500 independent realizations to make the result robustness. In all simulations, the total population is set to $N = 500$ and the average degree is $k = 6$.

To illustrate the spreading process, firstly we focus on the dynamics of informed individuals number n . n indicates the information diffusion range, and a larger n value in the stationary state indicates the broader spreading. For the spreading with the large λ is very efficient that almost all S nodes will be informed at the final state, there is no need to apply the link rewiring strategy to speed up the spreading process. Consequently, all the results in this Letter are obtained based on a relatively small λ ($\lambda = 0.2$) for discussing the spreading with link rewiring.

Figure 2 illustrates the dynamics of the informed population (n). It can be seen that the information spreading on scale-free networks is both broader and faster than that on the other networks with the same parameters. This might be caused by the heterogeneous degree distribution of the scale-free network where the hub node (large degree) will play an important role in the spreading process [29]. For the networks with mediate degree distribution, the number of the informed nodes follows the relation “random > small world > regular” which is coincide with the traditional understanding of the network epidemic spreading [30], but different from the information spreading model considering the social reinforcement [21]. For each kind of network, we show the results of $\beta = 2, 0, -2$ as well as the static network as the example to illustrate the influence of the link rewiring strategy on the information spreading process. The result obtained when $\beta = 2$ (the black cycle) is broader than others, and the enhancement is significant compared with the static network. $\beta > 0$ indicates that the active I nodes are more likely to rewire the links to nodes with more S neighbors, and the informed chance of the corresponding S nodes increases through the rewiring links. Once the large payoff S individuals know the information, the information will spread out rapidly with large probability for there are more S nodes connected with the large payoff nodes. In contrast, the spreading process will “die” quickly when $\beta < 0$. It is interesting to find that the influence of the link rewiring strategy on the scale-free network is also quite different from others. Neutrally rewiring strategy ($\beta = 0$) enhance the spreading on regular, small-world and random networks, while weaken the propagation effect on scale-free networks.

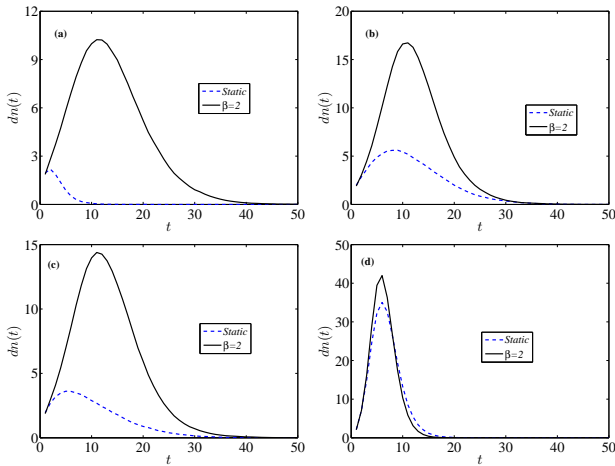


Fig. 4: (Color online) The dynamic of new informed individuals number ($dn(t)$) in each step. The green dash line and red line represent the static network and $\beta = 2$ respectively. (a) Regular networks; (b) Random networks; (c) Small-world networks; (d) Scale free networks. The samples are selected from 10^4 independent realizations.

considered as the cascade [31], which is a sequence of the informed individuals generated by the information spreading from the *seed*, and Fig. 3 gives the cascade size distribution from independent seeds. The cascade size distribution can be considered into two ranges: a power law for small size n and the rest part for large n , which is changed according to the network structure and β . From Fig. 3, it can be found that the stronger heterogeneity of the network degree distribution is, the more sharply the power law range decays and the more higher peaks the large n part exhibits. The cascade size distribution when $n > 10$ on the scale free network (the bottom four subgraphs) can be described as a log-normal function, which is very similar to the empirical result on *Digg*¹ where the information spreading on the fans’ network which also exhibits the power law degree distribution structure [32]. For the same network structure, there are more large size cascade for the link rewiring strategies when $\beta > 0$, resulting in more informed individuals in the stationary state which coincides with Fig. 2. It is interesting to find that the two ranges of the cascade size distribution are separated absolutely in the scale-free network with $\beta = 2$ (Fig. 3(c4)) and there is nearly no cascade with size ranging from 18 to 150, which indicates that the information will spread to a high level otherwise it will *die* quickly. The spread at the initial steps are very important, and if the information item can survive after the beginning several steps, it will spread out to a fraction of the total population.

In order to illustrate the evolution of the individuals cascade, we investigate newly informed individual number $dn(t)$ in Fig. 4. The results are averaging over the top 30% cascades that ranking with descending order of the cascade size in the final state, for the large number of the abortive spread processes will make the result become commonplace. For all the networks, the $dn(t)$ increases sharply at initial several steps, then decrease until the spread process stops. The $dn(t)$ with positive β is much larger than the corresponding static networks, which means that the information spreads much faster with the link rewiring strategy. In the scale-free networks, the spreading is quite quickly with the max $dn(t)$ is larger than 40, nearly 10% of the total population (500) are informed in one step.

The structure of small-world networks is parameterized by the randomly rewiring probability p_s , where $p_s = 0$ and $p_s = 1$ correspond to regular and random networks, respectively. We plot the number of the informed individuals at the stationary state as a function of p_s in Fig. 5. Similar to the previous studies [5, 33], we also find that the number of informed nodes in the stationary states increases with the small-world parameter p_s . In the small-world network, as the existence of the long range connection, the average distance of the network will decrease [27]. The information spreading should be faster and broader for the shorter distance between I and S nodes. The influence of the link rewiring with the Fermi function is robust

The informed individuals in the network can be con-

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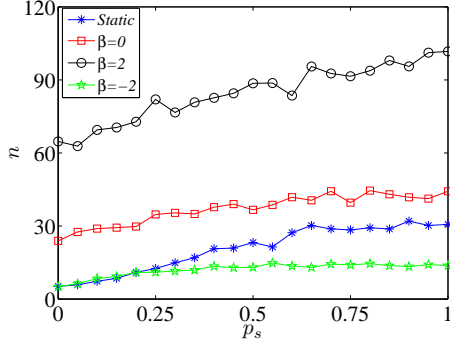


Fig. 5: (Color online) The number of the informed individuals as a function of parameter p_s with different methods including static network (blue pentagram), $\beta = 0$ (red square), $\beta = 2$ (black cycle) and $\beta = -2$ (green star).

with the change of p_s , and the information spreading is much broader with link rewiring when $\beta > 0$.

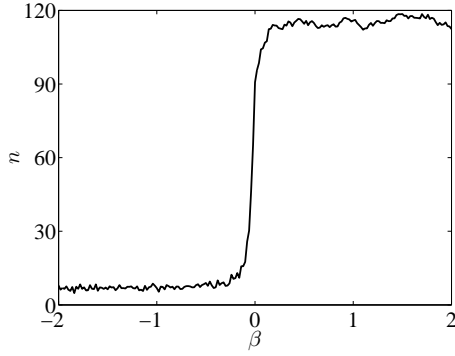


Fig. 6: (Color online) The fraction of the informed nodes as a function of the parameter β on scale-free network.

In order to illustrate how the parameter β influences the spreading process, we plot the number of informed individuals at the final state as a function of β on the scale free network in Fig. 6. The simulation result shows that a transition occurs between a regime where the spreading process dies out in a small neighborhood of the *seed*, and a regime where it spreads over a finite fraction of the whole population. The spread is very limited (the average cascade size is round 8) for $\beta < 0$, and it is very slightly that the increase of informed individuals number with the increasing of β when β is negative. It means that rewiring active links tends to connect the nodes with less S nodes connected will prevent information spreading. And this could be used as a strategy to control the spreading for virus and rumors. The spreading broadens very sharply when β increases around 0, which indicates that $\beta = 0$ should be the saltation point. When $\beta > 0$, the active link will be more likely to rewire to the nodes with more S neighbors. As long as β is positive, the information will spread into a very broad range (the average cascade size is larger than 110). The behavior of the informed numbers

round $\beta = 0$ indicates that the symbol rather than the accurate numerical value of β is the most significant factor with using the generalized Fermi function for choosing the rewiring probability.

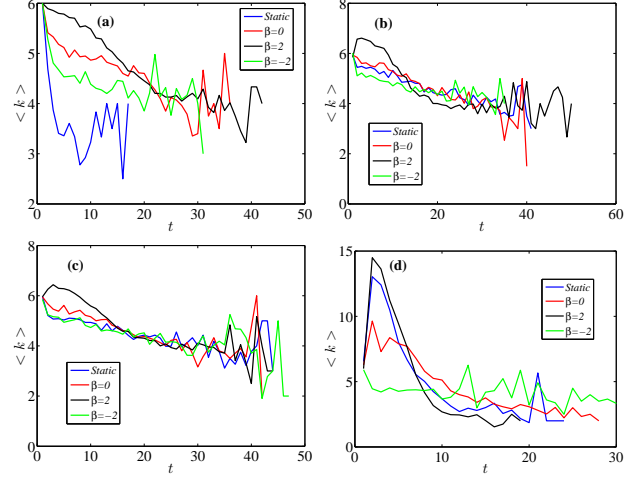


Fig. 7: (Color online) The average number ($\langle k \rangle$) of the S neighbors of the informed individuals at each time step with different methods including static network (blue), $\beta = 0$ (red), $\beta = 2$ (black) and $\beta = -2$ (green). (a) Regular networks; (b) Random networks; (c) Small world networks; (d) Scale free networks.

Discussion. — The simulation results show that the positive β induces a broader and faster information spread. To interpret the reason why the positive β enhance the spreading, we presents the dynamics of the average number of the S node connected with the new informed nodes in Fig. 7. The relationship between the four curves for the several time steps at the beginning of the spreading process coincides with the result in Fig. 2, which means that the information item will spread broader and faster if the nodes with more S neighbors are informed when the spreading started. The result on the scale-free network is consistent with Barthélemy's work [29], where the dynamical of the spreading is characterized by a hierarchical structure, which the information is informed with large degree nodes firstly, then pervades the network via the smaller degree classes rapidly. And the hierarchical spreading patten would be more obvious when $\beta > 0$ (the black curve in Fig. 7(d)). However, the large-degree nodes are not always speed up the spreading process, such as the game-theoretic models of the innovation spread [34].

For the positive β on the generalized Fermi function, we will obtain the large rewiring probability $\frac{1}{1 + e^{\beta(\pi_j - \pi_{j'})}}$ if the payoff of S -state node j' is larger than j , and vice verse. That is to say, the active I nodes will be more like to rewire the link to the S nodes with more S neighbors. Following this process, the nodes that have more S neighbors have more chance to be informed, and the information could be more likely to spread out through such nodes. Therefore,

we obtain more average uninformed neighbors of the new informed individuals with positive β (Fig. 7). When $\beta < 0$, the active I individuals will more likely reconnect to the S individuals with less S neighbors, leading to the quick annihilation of the spreading.

Conclusion. — In this Letter, we proposed a dynamic model for the information spreading process that considers the link rewiring based on the simple susceptible-infected (SI) model. Unlike the traditional SI model, each $S - I$ link just can be used once in our model for the non-redundancy of contacts. The rewiring probability is chosen following the generalized Fermi function based on the payoff between the two selected uninformed individuals. The simulation result shows that the information will spread broader and faster when the parameter $\beta > 0$ because that the uninformed individuals with more uninformed neighbors are more likely to be informed at the beginning spreading steps with the positive β . Through these uninformed hubs, the information item can spread over a finite proportion of the population quickly. The cascade size distribution indicates that the spread at the initial steps are very important, where the information can spread out to a finite fraction of the total population if it still survives after the beginning several steps. In addition, the negative β can be used as a strategy to control the spreading for virus and rumors.

Recently, the research of the information spreading based on temporal networks has attracted more and more attention [35,36]. Simulation results in this Letter demonstrated that with the large payoff trend, the rewiring strategy can enhance the information spreading process. Moreover, it is found that the human communication pattern is of critical importance in the information diffusion [16]. For a more detailed evaluation, the temporal patterns of human communication such as the bursty activity should be studied on the rewiring strategy in future work.

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